A MULTISCALE HYBRID METHOD FOR MATERIALS CONTAINING DEFECTS AND INHOMOGENEITIES

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The Finite Element Method (FEM) requires full-domain discretization and is therefore more suitable for problems with comparable characteristic length scales. When applied to multiscale problems, FEM becomes extremely inefficient or even practically impossible, as the number of discretized elements can be prohibitively large. In this work, a Multiscale Hybrid Method (MHM) is developed for the micromechanics of materials containing defects and inhomogeneities of different length scales. In MHM, only the macro boundary and the inhomogeneities need to be discretized. Its micro-macro integral formulation allows the multiscale problem to be, in effect, decomposed into two associated subproblems. At the macro-level, the problem boundary is discretized, and the resulting formulation is equivalent to that of the boundary element method; at the micro-level, only the inhomogeneities are discretized, and the resulting formulation resembles that of the Eshelby's inclusion problem [1]. Interactions between the inhomogeneities and the boundary are provided explicitly in the governing equation, while the inhomogeneity length scale is fully decoupled from the boundary scale, offering the capability to handle essentially different length scales. Different from Eshelby's approach in which strains and stress are the basic variables, the displacements are taken as the basic variables in the present method. The displacement-based approach lowers the order of singularity in the governing integral equations and offers high accuracy at reduced computational cost. A coordinate origin shift scheme is used to remove the weak singularity in the displacement integral equation, while the subtraction technique and the Eshelby's method are employed to remove the strong singularity in the calculation of stresses inside the inhomogeneities. The method can handle arbitrary geometry and general loading conditions at the macro scale and the interaction of microstructural features at the micro scale, and offers orders of magnitude of increased efficiency over FEM.

General formulas for 2D and 3D problems were derived, while a 2D version was implemented in a software package. Several tests for an elastic body with inhomogeneities were investigated, and the results were compared with closed form solutions or FEM. The effects of elastic modulus ratio and boundary-inhomogeneitiy interaction were also studied. The simulation demonstrated the potential, efficiency and versatility of the proposed MHM.

References

[1] J.D. Eshelby, 1957, The determination of the elastic field of an ellipsoidal inclusion and related problems, *Proc. R. Soc. Lond.*, **A241**, 376-396